Assessing Key Understandings in Mathematics

Introduction

> *A school's assessment system could assess everything students are learning, but then teachers would spend more time assessing than teaching. The important point here is that any assessment system needs to be selective about what gets assessed and what does not, and so the assessment system needs to focus on the 'big ideas' in each curriculum area…You can't assess everything – be selective.* Dylan William (2014)

The 2014 National Curriculum for Mathematics is packed with individual objectives within the programmes of study. Many assessment systems include all of these, leading to an expectation that teachers will assess against every single objective for every child. There are a number of dangers with this approach, beyond the obvious concern over teacher workload:

- The programmes of study do NOT fully reflect the aims of the National Curriculum. These aims are intended to underpin all mathematics:

> *The aims of the National Curriculum should be integral to teaching. When developed effectively, they are the key characteristics of good and outstanding practice. Teaching that focuses heavily on covering the listed content, without developing understanding, reasoning and problem solving at the same time is missing the strong drive that the aims represent for improving mathematical education. Such teaching is likely to require improvement.*
>
> Ofsted 2015

Therefore it is essential that assessment in maths takes account of and reflects the intention of the aims; this will not be achieved by focussing assessment on the programmes of study alone.

- There is no indication of the relative importance of different programmes of study within the National Curriculum, so each element may be assigned the same 'weight' even though some have more depth and importance than others.
- Key ideas in mathematics are spread across different domains. Assessing the elements individually will not necessarily indicate that a child has made crucial connections and understood how the maths fits together.

For these reasons we have written the **Key Understandings** for each year group from Y1 to Y6. These identify 'big ideas' or 'key constructs' which children need to understand each year and explicitly reflect the aims of the National Curriculum. The intention is that these will be used by teachers to make judgements at the end of each year; during the year teachers will use assessment within teaching sequences to decide if children are on track to achieve the key understandings by the end of the year. (For further information on assessment within teaching sequences, see the paper *Elicitation Tasks in Mathematics*.)

Key Understandings Explained

The key understandings identified in each year group come from all areas of the curriculum and are built around the themes of Number Sense, Additive Reasoning, Multiplicative Reasoning and Geometric Reasoning. Important things to note are as follows:

- **'Represent and explain'** – many of the understandings contain this expectation. Representation comes in a variety of forms:
    - Mathematical images/pictures – whilst structured images can be used to support understanding, they are also a key element for children demonstrating their understanding. It is often through unpacking thinking so that it can be exemplified through the use of different manipulatives, pictures and diagrams that children show the depth of their understanding of the structure of the mathematics and reveal misconceptions.
    - Language – there is an emphasis on talk within the aims of the National Curriculum and children need to be provided with the opportunity to share their thinking and craft their responses. The use of mathematical language will be important but it is not about focussing on vocabulary in isolation, it is about constructing meaningful explanations which are structured to communicate clear understanding. Individual words alone do not demonstrate understanding. Sometimes the language used in the key understandings should also be used by the children but this is not always the case; for example in the teaching guidance for Y4 it refers to the associative and distributive laws. The important thing here is that the children understand how these laws work and use this understanding when solving problems, not that they can remember the names for them.

- Context – understanding of solutions to problems is required when these solutions are put into a context; context forces children to go beyond 'doing' the maths. For example, a child might calculate 2 ÷ 3 = $^{2}/_{3}$ but when put into the context of two chocolate bars being shared between three people, they might suggest that each person gets $^{2}/_{3}$ of two bars, revealing a misconception. Context can also make the maths meaningful. The expectation is that understanding in number sense, additive reasoning and multiplicative reasoning will be demonstrated within the contexts of measures and statistics as well as other contexts. Measures include money and time; for example, in Y4 time could be a context within both additive reasoning (conversion of 12 hour to 24 hour) and multiplicative reasoning (conversion of hours to minutes)
- Symbols – this includes mathematical symbols (such as <, >, = etc.) and words (such as number names).

It is intended that children can represent their understanding in all of these ways, can explain and connect the different ways and use their understanding to make decisions.

- **'…appropriately choosing from and using what they know and understand'** features in all of the understandings related to calculation. This is a key aspect of mathematics, the expectation that children will be flexible and make decisions about how to solve a problem, based on their own understanding and the numbers involved. In order to support teachers with this key area, we have written the companion paper '*Teaching guidance for 'choosing from and using"*. This sets out, for both additive reasoning and multiplicative reasoning, the understanding and knowledge that children should be using when making decisions within each year group. In order for children to demonstrate their use of the knowledge and understanding listed, they will need to encounter problems which are best solved in different ways. For example, solving 'I have £387 and spend £80' involves using understanding of place value whilst solving 7 = ? – 9 could involve using understanding of equivalence, the relationship between addition and subtraction and number facts. Not only do children need to be making decisions, they also need to be able to explain and justify their decisions. This will include checking and estimating.

- **Mentally –** this is used to contrast with a formal written algorithm. Working things out 'mentally' may include making marks on paper or manipulating resources where the drawings/symbols/images are being used to capture thinking and aid problem-solving, rather than for 'doing' the maths.
- **Generalising** - in order for children to demonstrate they have generalised understanding, they may need to go beyond the size of numbers set out for their year group in the curriculum. For example, by the end of Y1, children who are demonstrating their understanding of one more and one less will do this for numbers beyond the teens and beyond two-digit numbers.
- **Composition of numbers** – understanding the composition of numbers is critical for building number sense and a flexible approach to calculating. This starts with understanding the additive composition of small numbers in Y1, for example 4 = 3 + 1, 4 = 2 + 2 etc. This shifts the focus from knowing a list of calculations, or number bonds, to understanding how a number can be made and the relationship between these different ways. This means that rather than seeing maths as a memory test, children are building relational understanding which is key to fluency.

  Parish, drawing from Fosnot and Dolk (2001) defines fluency as 'knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems.' (Parish 2014, p 159). Whether or not we believe that fluency requires more than the recall of math facts, research evidence points in one direction: The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense. Boaler 2014

  This develops into the multiplicative composition of numbers which links to factors and multiples, for example understanding:

  24 = 6 x 4 = 8 x 3 = 12 x 2 = 2 x 2 x 6 etc.

Judgements against the Key Understandings can be recorded in any way to suit the school; Ofsted does not expect to see any particular system of assessment in place.

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Expectations

> *The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.* National Curriculum (2013)

The Key Understandings reflect the expectations that children will understand key elements of mathematics by the end of each year. Sometimes, maths is introduced in a specific year group in the National Curriculum but does not appear in that year group in the Key Understandings, either because it is expected that understanding is secured during the following year or because we have identified mathematics which needs to be secured first and so moved the expectation to the following year. This is in line with the way that the curriculum has been set out. For example, whilst quarters appear in Y1 in the National Curriculum, they appear in the Key Understandings in Y2. This is because we think that securing a deep understanding of doubling and halving, in a variety of contexts, is more important in Y1, so that this provides a solid foundation for developing understanding of fractions.

Moderation

The use of the key understandings for making judgements will allow teachers to moderate at the end of the year. To assist with the moderation process, and to build a picture of maths across the school, it is useful for subject leaders to talk to children in the different year groups; children are the best evidence in mathematics and so talking to children can provide clear evidence as to whether they understand the key ideas for their year group.

References

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Key Understandings - Y1

- Pupils represent and explain what happens when counting forwards and backwards in ones and how they know one more or less than any given number, in different contexts (including measures)
- Pupils represent and explain how to distinguish between 'teen' and 'ty' numbers
- Pupils represent and explain how numbers up to ten can be composed and decomposed in different contexts (including measures)
- Pupils represent and explain addition and subtraction problems in different contexts. They solve these problems by taking account of the numbers involved, appropriately choosing from and using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils count fluently in twos, fives and tens in different contexts, explain what is happening when counting and explain how they know which numbers are multiples of two, which are multiples of five and which are multiples of ten.
- Pupils represent and explain what happens when doubling and halving in different contexts (including measures) and use this understanding to solve problems.
- Pupils recognise and identify shapes in different orientations in their environment, describe what is the same and what is different about them and justify their thinking:
    - 2D – rectangles (including squares), triangles and circles
    - 3D – cuboids (including cubes), pyramids and spheres

*see teaching guidance for additional details

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Key Understandings - Y2

- Pupils count fluently forwards and backwards in tens, starting at any one- or two-digit number, and represent and explain how they know ten more or less than any number under 100, in different contexts (including measures). They count fluently in threes in different contexts and explain what is happening when counting.
- Pupils represent and explain how their understanding of composing and decomposing numbers up to ten can be extended to larger numbers in different contexts (including measures)
- Pupils compare, represent and explain the value of two-digit numbers in different contexts (including scales). They place and identify these numbers on a number line, using their understanding of how close the numbers are to decade numbers, explaining and justifying their decisions.
- Pupils represent and explain commutativity, the relationship between subtraction and addition and subtraction as 'take away', 'difference' and 'how many more to make' in different contexts (including measures and statistics).
- Pupils represent and explain addition and subtraction problems involving pairs of two-digit numbers, in different contexts (including measures and statistics). They solve these problems mentally by taking account of the numbers involved, appropriately choosing from and using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils represent and explain the difference between odd and even numbers and use this understanding to identify large multiples of two.
- Pupils represent and explain commutativity and the relationship between multiplication and division in different contexts.
- Pupils represent (including with arrays) and explain multiplication and division problems (involving 2s, 5s and 10s) in different contexts (including interpreting data and time). They solve these problems by appropriately choosing from and using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils represent and explain how to find halves and quarters and the relationship between them, in the context of discrete objects, continuous measures, shapes, movement (turn) and time and use this understanding to solve problems.
- Pupils choose and use appropriate standard units (cm or m, g or kg, ml or l, C^0) and equipment to estimate and measure in different contexts, explaining and justifying their decisions.
- Pupils recognise and identify 2D and 3D shapes in different orientations in their environment and explain the properties of the shapes:
    - 2D – number of sides, length of sides and line symmetry
    - 3D – shape of faces and number of faces, vertices and edges

*see teaching guidance for additional details

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Key Understandings - Y3

- Pupils represent and explain the base ten structure of the number system (up to thousands).
- Pupils place and identify numbers (up to one thousand) on a number line (including scales in measures and statistics) using their understanding of how close the numbers are to key boundaries, explaining and justifying their decisions.
- Pupils represent and explain addition and subtraction problems involving three-digit numbers in different contexts (including extracting information from graphs, charts, tables and measuring scales). They solve these problems by taking account of the numbers involved, appropriately choosing to use mental or written methods, using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils represent and explain, in appropriate contexts: multiplication as both repeated addition and scaling (including fractions) and division as both sharing and grouping.
- Pupils represent and explain multiplication and division problems (involving 3s, 4s and 8s) in different contexts (including statistics). They solve these problems by taking account of the numbers involved, appropriately choosing from and using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils represent and explain a unit fraction as the result of a division in an appropriate context and how they know that, for unit fractions, as the denominator increases the size of the number decreases. They place and identify unit fractions on a number line, explaining and justifying their decisions.
- Pupils represent and explain angle as a measure of turn.
- Pupils explain the properties of 2D shapes related to the angles (acute, obtuse and right angles), the sides (horizontal, vertical, perpendicular and parallel lines) and the perimeter and draw shapes to demonstrate their understanding.

*see teaching guidance for additional details

Key Understandings - Y4

- Pupils represent and explain how the multiplicative nature of the number system includes decimal numbers (up to two decimal places) in different contexts. They place and identify these numbers on a number line (including scales in measures and statistics) using their understanding of how close the numbers are to key boundaries, explaining and justifying their decisions and using this understanding to round numbers.
- Pupils represent and explain addition and subtraction problems involving four-digit numbers in different contexts (including extracting information from graphs, charts, tables and measuring scales). They solve these problems by taking account of the numbers involved, appropriately choosing mental or column methods using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils represent and explain a fraction as the result of a division in an appropriate context.
- Pupils place and identify non-unit fractions on a number line, explaining and justifying their decisions.
- Pupils represent and explain how numbers (within the multiplication tables) can be multiplicatively composed and factorised in different contexts (including measures).
- Pupils represent and explain multiplication (involving up to three-digit numbers by one-digit numbers) and division problems in different contexts (including measures). They solve these problems by taking account of the numbers involved, appropriately choosing from and using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils draw, describe and explain the properties of different triangles and quadrilaterals, including angles, sides, lines of symmetry and perimeter.
- Pupils explain how to locate points on a grid in the first quadrant and use this knowledge and understanding to solve problems.

*see teaching guidance for additional details

Key Understandings - Y5

- Pupils represent and explain how the multiplicative nature of the number system includes whole numbers beyond thousands and decimal numbers beyond hundredths, in different contexts. They use this understanding to convert between different units of measures, explaining and justifying their decisions.
- Pupils place and identify whole numbers (beyond thousands) and decimal numbers (beyond hundredths) on a number line (including scales in measures and statistics) using their understanding of how close the numbers are to key boundaries, explaining and justifying their decisions and using this understanding to round numbers.
- Pupils represent and explain how the number system includes negative numbers. They place and identify negative numbers on a number line, explaining and justifying their decisions.
- Pupils represent and explain equivalent fractions and fractions bigger than one. They place and identify these fractions on a number line, explaining and justifying their decisions.
- Pupils represent and explain addition and subtraction problems involving numbers with more than four-digit numbers in different contexts (including extracting information from graphs, charts, timetables and measuring scales). They solve these problems by taking account of the numbers involved, appropriately choosing mental or column methods using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils represent and explain multiplication and division problems (involving up to four-digit numbers by one-digit numbers) in different contexts (including measures). They solve these problems by taking account of the numbers involved, appropriately choosing mental or formal written methods and using what they know and understand*, explaining their decisions and justifying their solutions.
- Pupils represent and explain perimeter and area in relation to rectangles and use this understanding and their understanding of calculation, to solve problems involving rectilinear shapes, explaining and justifying their decisions.
- Pupils draw, measure, identify and explain angles (including angles at a point on a straight line, one whole turn and reflex angles) and use their understanding of angle to describe the properties of different shapes (regular and irregular).
- Pupils explain how to reflect and translate shapes on a grid in the first quadrant and use this knowledge and understanding to solve problems.

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Key Understandings – Y6

- Pupils represent and explain the relationship between decimals, fractions, percentages and ratio and use their understanding to solve problems.
- Pupils represent and explain addition and subtraction problems involving fractions with different denominators, decimals (beyond two decimal points) and calculating the interval across zero in different contexts (including extracting information from graphs, charts, timetables and measuring scales). They solve these problems by taking account of the numbers involved, appropriately choosing mental or column methods using what they know and understand*, explaining their decisions and justifying their solutions and level of accuracy.
- Pupils represent and explain multiplication, division and ratio problems (including up to four-digit numbers by two-digit numbers, fractions and decimals) in different contexts (including converting between metric and imperial measures). They solve these problems by taking account of the numbers and their properties (square, prime common multiples etc.) involved, appropriately choosing mental or formal written methods and using what they know and understand*, explaining their decisions and justifying their solutions and level of accuracy.
- Pupils represent and explain multi-step problems involving addition, subtraction, multiplication and division in different contexts (including finding the mean). They solve these problems by taking account of the numbers involved and the order of operations, explaining their decisions and justifying their solutions
- Pupils represent and explain how to find the volume of cubes and cuboids and use their understanding of properties of shapes (including circles), area and volume to solve problems.
- Pupils represent and explain positions on a grid with four quadrants and how to reflect and translate shapes and use this knowledge and understanding to solve problems.
- Pupils recognise 3D shapes represented in different ways (including as 2D drawings and nets) and can draw accurate 2D shapes using given information (including to form nets) explaining and justifying their thinking.
- Pupils explain the use of letters to represent relationships, variables and unknowns in familiar additive, multiplicative and geometric situations and use their understanding to solve problems involving letters.

*see teaching guidance for additional details

Teaching Guidance for 'choosing from and using'

The expectation is that as children move through the primary years they will have an increasing mathematical understanding and bank of known facts which they access when making decisions about how to solve a problem. Children's choices when calculating should reflect their understanding of additive and multiplicative relationships, make use of what they know and be appropriate for the numbers involved. They should have a sense of the size of the missing number and recognise when their solution cannot be correct.

Teachers will need to ensure that children tackle a variety of problems which involve unknown numbers in different places and require different decisions. To help plan for and assess this, the knowledge and understanding which children should be choosing from and using to solve problems, for both additive reasoning and multiplicative reasoning, is set out below for each year group.

	Additive Reasoning	**Multiplicative Reasoning**
Y1	• number facts • understanding of equivalence • understanding of one more and one less. • understanding the effect of adding and subtracting 0	• understanding doubling and halving • counting in 2s, 5s and 10s • understanding of equivalence
Y2	• number facts • understanding of equivalence • understanding of place value • understanding of commutativity of addition • understanding of the structures of addition and subtraction and the relationship between them	• understanding doubling and halving • counting in 2s, 5s and 10s • understanding of equivalence • understanding of the array • number facts • understanding of commutativity of multiplication • understanding of multiplication and division and the relationship between them
Y3	• number facts • understanding of equivalence • understanding of place value • understanding of commutativity of addition • understanding of the structures of addition and subtraction and the relationship between them	• understanding doubling and halving • understanding of equivalence • understanding of the array • number facts • understanding of commutativity of multiplication • understanding of the structures of multiplication and division and the relationship between them • relationships between multiples of 2, 4 and 8 • understanding of place value
Y4	• number facts • understanding of equivalence • understanding of place value • understanding of commutativity of addition • understanding of the structures of addition and subtraction and the relationship between them	• understanding doubling and halving • understanding of equivalence • understanding of the array • number facts, including factor pairs • understanding of commutativity of multiplication • understanding of the structures of multiplication and division and the relationship between them • relationships between multiples • understanding of place value • understanding of fractions and the relationship between fractions, multiplication and division • understanding of multiplication as area of a rectangle (underpinned by the array) • understanding of the associative and distributive laws • understanding the effect of multiplying by 0 and 1 and dividing by 1

Teaching Guidance for 'choosing from and using'

	Additive Reasoning	Multiplicative Reasoning
Y5	• number facts • understanding of equivalence • understanding of place value • understanding of commutativity of addition • understanding of the structures of addition and subtraction and the relationship between them • understanding of fractions, including mixed numbers and improper fractions • understanding of equivalent fractions involving denominators which are multiples of the same number	• understanding doubling and halving • understanding of equivalence • understanding of the array • number facts, including factor pairs • understanding of commutativity of multiplication • understanding of the structures of multiplication and division and the relationship between them • relationships between multiples • understanding of place value • understanding of fractions and percentages and the relationship between them and multiplication and division • understanding of multiplication as area of a rectangle (underpinned by the array) • understanding of the associative and distributive laws • understanding the effect of multiplying by 0 and 1 and dividing by 1 • understanding of primes, composites, squares and cubes
Y6	• number facts • understanding of equivalence • understanding of place value • understanding of commutativity of addition • understanding of the structures of addition and subtraction and the relationship between them • understanding of fractions, including mixed numbers and improper fractions • understanding of equivalent fractions and common multiples • understanding of negative numbers	• understanding doubling and halving • understanding of equivalence • understanding of the array • number facts, including factor pairs • understanding of commutativity of multiplication • understanding of the structures of multiplication and division and the relationship between them • relationships between multiples • understanding of place value • understanding of fractions, percentages and ratio and the relationship between them and multiplication and division • understanding of multiplication as area of a rectangle (underpinned by the array) • understanding of the associative and distributive laws • understanding the effect of multiplying by 0 and 1 and dividing by 1 • understanding of primes, composites, squares and cubes • understanding of mean • understanding of equivalent fractions